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I. INTRODUCTION

At the last symposium Schaaf and Maslach (1963) presented experimental cylinder drag results which were obtained at sufficiently high Knudsen numbers and with small enough scatter in the data to make a meaningful comparison with the theories then available (Lunc and Lubonski, 1956; Baker and Charwat, 1958; Willis, 1959) for high Knudsen number flows. A large discrepancy between theory and experiment was noted at that time. As theoretical results were not then available for a cylinder the comparison was based on results for a two-dimensional strip normal to the flow. However, it was felt that this geometric effect would not significantly change the qualitative nature of the comparison.

Since the last meeting the following steps have been taken in an attempt to reconcile the results of theory and experiment. Firstly, new experimental results have been obtained (Section II). Secondly, theoretical calculations have been performed for the cylinder (Taub, 1964). Thirdly, a method different from that of Schaaf and Maslach (1963) has been used to relate the Knudsen number to the parameter which arises in those theoretical calculations where a statistical model is used to represent the intermolecular collision process. This is discussed in Section III.

II. NEW EXPERIMENTAL RESULTS

Since 1962 a continuing program of drag force measurement in the near free molecule and free molecule regimes has been carried out at Berkeley extending the range of flow conditions to higher Mach numbers and utilizing both cylindrical and flat strip models. Essentially, the same experimental methods have been used as were previously described (Maslach and Schaaf, 1963) with an improved balance mechanism, Fig. 1, and the use of free jet testing techniques to achieve higher Mach numbers at lower densities. The extensive precautions taken to determine precisely the characteristics of the flow fields issuing from these simple convergent nozzles are the subject of a separate paper (Ashkenas and Sherman, 1964).

Cylinder drag data were obtained at Mach numbers of approximately 6, 8, and 10 with a range of Knudsen numbers from approximately 1 to 32 (Tang, 1964). Flat strip drag data for the model normal to the flow were obtained at approximately the same Mach numbers with a similar range of Knudsen numbers (Ko, 1964).

All results approach diffuse free molecule flow limiting conditions for high Knudsen numbers; typical data are shown in Figs. 2 and 3. Previous results at Mach approximately 5.92 using uniform flow field testing techniques were essentially duplicated, the maximum discrepancy being 2.8%, utilizing the free jet testing methods, Fig. 4.

III. RELATION OF KNUDSEN NUMBER TO COLLISION PARAMETERS

All calculations to date for external aerodynamic problems have used either perfectly elastic spheres to represent the collision process (Lunc and Lubonski, 1956; Baker and Charwat, 1958) or replaced the Maxwell-Boltzmann collision operator with a statistical type model (Willis, 1959, 1960; Taub, 1964; Rose, 1964).

For hard sphere molecules the mean free path is given unambiguously by

$$\lambda = (\sqrt{2}\,\pi\, n\, \sigma^2)^{-1} \tag{1}$$

and the (Chapman-Enskog) viscosity is given by

$$\mu = \rho\,\lambda\,(2kT/\pi\, m)^{1/2} \tag{2}$$

Two statistical models have been used. Willis (1959 and 1960) and Taub (1964) used the "modified Krook" model. This model was specifically designed for nearly free molecular conditions. For convenience a brief description is given in the Appendix. Rose (1964) uses the original Krook model (Bhatnager, et.al., 1954)

Nearly free molecular analyses using either of these models gave results in terms of a "natural" small parameter $\alpha = \delta n_\infty (D/2)(m/2kT_b)^{1/2}$. In this relation D is a typical body dimension, n_∞ the number density at infinity, δ is a constant appearing in the model (such that $1/\delta n_\infty$ is the relaxation time for the translational degrees of freedom), and T_b is the temperature characterizing the Maxwell distribution which the molecules reflected from the body are assumed to possess.

In comparing theory with experiment we must relate α to the free stream Knudsen number. Schaaf and Maslach (1963) used the following relation:

$$\alpha = 0.6\ S_b / Kn_\infty \tag{3}$$

where $S^2 = m\, U_\infty^{\ 2}/2kT$ and $Kn_\infty = \lambda_\infty / D$. This relation is empirical and was obtained by a comparison of high Mach number strip drag results using the modified Krook model and hard sphere molecules.

A very different result is obtained if we choose to define mean free path from the viscosity formula of Eq. (2). This formula is only valid for near equilibrium conditions, i.e., far from the body. For the simple Krook model we have the result

$$\mu_\infty = kT_\infty / \delta \tag{4}$$

and hence

$$\alpha = (\pi^{1/2}/4)(T_\infty / T_b)^{1/2} / Kn_\infty \tag{5}$$

There will be a large difference between the results of Eqs. (3) and (5) if the speed ratio, S_∞, is not close to unity. For the modified Krook model the meaning of the viscosity is not so clear as the model is specifically designed for nonequilibrium situations. However, we propose to use the relation in Eq. (5) for this case also.

There are, of course, several other ways in which α could be related to Kn_∞. Both the physical arguments underlying the first collision methods and order of magnitude analyses using the first iterate method with the model equations suggest that the main source of correction to the free molecular drag, etc., comes from collisions between molecules leaving the body and free stream molecules. A relation between α and Kn_∞ might then be developed by studying the rate of such collisions. This will not be attempted here.

IV. COMPARISON BETWEEN THEORY AND EXPERIMENT

Unless specifically stated otherwise we use Eq. (5) to relate α and Kn_∞ for those theoretical results that use the statistical models.

A. Cylinder and Two-Dimensional Strip Drag

The theoretical results, based on the modified Krook model, are of the form

$$C_D = C_{Dfm} + H(S_\infty, S_b)(\alpha \ln \alpha) \qquad (6)$$

The method used is that of integral iteration (Willis, 1959). The function H is given in Table I for typical values of S_∞ and S_b used in the drag experiments. It should be noted that terms of order α have been neglected compared to those of order $\alpha \ln \alpha$. This naturally limits the range of validity of the theory significantly. Typical comparisons of the data with the theoretical results are shown in Figs. 3 and 5. For the cylinder we show the effect of the two interpretations of the α-Kn_∞ relation. It is obvious that the agreement is far superior using the mean free path based on viscosity, i.e., Eq. (5).

B. Cylinder Equilibrium Temperature

The cylinder equilibrium temperature can be calculated from results given by Taub (1964) for the heat transfer corrections. The calculation must be performed using the value of S_b, uniquely determined by S_∞, which corresponds to zero heat transfer for the free molecular flow. Using the modified Krook model we obtain the following result for the normalized equilibrium temperature

$$\eta^* = \frac{T_{eq} - T_{eq}(Kn_\infty = 0)}{T_{eq}(Kn_\infty = \infty) - T_{eq}(Kn_\infty = 0)} = 1 + (\alpha \ln \alpha) G(S_\infty, S_b) \tag{7}$$

Assuming a monatomic gas, a recovery factor of unity in continuum conditions, and supersonic flow, we have approximately

$$T_{eq}(Kn_\infty = 0)/T_{eq}[Kn_\infty = \infty] = (1 + 0.4\, S_\infty{}^2)(T_\infty/T_{eq}[Kn = \infty]) \tag{8}$$

Using this result and the standard formula for $(T_\infty/T_{eq}[Kn=\infty])$ for a cylinder (Schaaf, 1963) we obtain

$$\begin{aligned} S_\infty &= 2, & S_b &= 1.12, & G &= 0.190 \\ S_\infty &= 4, & S_b &= 1.31, & G &= 0.304 \\ S_\infty &= 5, & S_b &= 1.35, & G &= 0.340 \end{aligned} \tag{9}$$

These results are plotted in Fig. 6 using Eq. (5) to relate α to Kn_∞. The data are taken from Dewey (1961). Also shown is the result given by Schaaf and Maslach (1963), namely

$$\eta^* = 1 - \frac{0.86}{Kn_\infty} (\ln Kn_\infty) \tag{10}$$

which was obtained from results in (Willis, 1959) using hard sphere molecules with S_∞ approaching infinity. The modified Krook results fall within the scatter bounds of the data for $Kn_\infty \gtrsim 5$. While this agreement is scarcely good, it is far better than the gross discrepancy between the result of Eq. (10) and the data

The value of G is quite sensitive to the assumption regarding the recovery factor under continuum conditions and may be expected to vary significantly for diatomic gases. There is also some question regarding the Prandtl number implied by the model.

C. Sphere Drag

Sphere drag has been predicted by first collision (Baker and Charwat, 1958) and first iterate (Willis, 1960) methods. Recently Rose (1964) derived an expression for the drag by a much different analytical method involving the use of the linearized Krook equation and Fourier transform techniques. All the above results were for large free stream Mach number and $S_b \gtrsim 2$. Rearranging all the results in terms of Kn_∞, we find*

$$C_D - C_{Dfm} = -(0.24\, S_b + 1.06)/Kn_\infty \qquad \text{(Baker and Charwat) (11a)}$$

$$= -(0.165\, S_b + 1.44 - 1.13/S_b)/(S_\infty Kn_\infty) \qquad \text{(Willis) (11b)}$$

$$= -(0.33\, S_b - 0.12)/S_\infty Kn_\infty \qquad \text{(Rose) (11c)}$$

Equation (11b) is obtained by fitting numerical results for $S_b \geq 2$.

We have compared the theories with typical data given by Kinslow and Potter (1963) in Fig. 7. (We have plotted mean values when more than one measurement was made at the same nominal conditions.) The data were obtained for $S_\infty = 8.8$ and $S_b \geq 4.4$, so all the theories should be applicable.

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*Formula (11a) is slightly different from that given by Schaaf and Maslach (1963) due to a difference in expressing one of Baker and Charwat's parameters in terms of Kn_∞ and S_b. Equation (11b) corrects a typographical error in a corresponding equation in (Willis, 1960).

It can be seen that the results based on hard sphere molecules (Eq. 11a) are again in serious disagreement with the data The results for both models, however, are in quite good quantitative agreement with the data. These remarks hold true for all of Kinslow and Potter's data with $S_b = 6.25$ (Fig. 7), 5.85, 5.0, and 4.4. The relative difference between the results of Eqs. (11b) and (11c) increases as S_b decreases, but values of S_b significantly lower than 4 4 will be needed to discriminate between the results.

V. CONCLUSIONS

1) New cylinder and flat strip drag data for models normal to the flow have been obtained utilizing free jet testing techniques. For similar Mach numbers the data essentially duplicate previous work using uniform flow field testing techniques.

2) For the drag of a cylinder or two-dimensional strip the data have been obtained for sufficiently high Knudsen number and with sufficiently low scatter to provide a test of the theoretical predictions.

3) The theoretical results obtained from the first iterate method, using the modified Krook model to represent the collision process, and determining the mean free path from viscosity in the gas far from the body, are in reasonable agreement with experimental data for cylinder, two-dimensional strip, and sphere drag. For the cylinder equilibrium temperature the agreement is only fair, but at least $(T_{eq}-T_{eq}[Kn_\infty=\infty])$ has the correct order of magnitude for $Kn_\infty \gtrsim 5$.

4) Results obtained using the linearized Krook model (Rose, 1964) are also in reasonable agreement with the sphere drag data of Kinslow and Potter (1963).

5. Theoretical results using hard sphere molecules to represent the collision process do not agree with the above cited experimental data.

6. A more definitive comparison could be obtained if the theoretical calculations for the two-dimensional bodies were extended to include the terms of order $(1/Kn_\infty)$ as well as $(1/Kn_\infty) \ln (Kn_\infty)$, and if the sphere drag results could be obtained at higher Knudsen numbers and for a wider range of body to free stream temperature ratios.

TABLE I

Values of $H(S_\infty, S_b)$ (Eq. 6)

M_∞	S_∞	S_b	H(cylinder)	H(strip)
1.96	1.64	1.17	1.218	1.644
4.00	3.34	1.55	1.162	1.666
5.92	4.95	1.66	1.148	1.660
9.85	8.24	1.71	1.166	--
10.09	8.44	1.69	--	1.656

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APPENDIX

THE MODIFIED KROOK MODEL

The simple Krook model (Bhatnager et. al., 1954) considers all collisions at a point statistically and makes no distinctions between the various types of collisions that occur. This appears, intuitively, to be too simple a representatio for a nearly free molecular flow where the distribution function has very different properties depending on whether or not the molecular velocity is within the solid angle subtended by the body and directed away from the body. As a step toward the Maxwell-Boltzmann type of collision operator, where all possible types of collisions are considered, we propose the following model, specifically designed for nonequilibrium conditions. The molecules at any point $\underline{r}$ are divided into two classes. Those whose molecular velocities $(\underline{\xi})$ lie in the outward drawn solid angle subtended by the body are called class b and all other molecules are called class c. The collision term becomes

$$\left(\frac{\partial f_b}{\partial t}\right)_{\text{collisions}} = -\delta f_b(n_b w_{bb}+n_c w_{bc}) + \delta(n_b^2 w_{bb}\Phi_{bb}+2n_b n_c w_{bc}\Phi_{bc}+n_c^2 w_{cc}\Phi_{cc}) \qquad \text{(A1)}$$

where δ is a constant, w_{bb}, $w_{bc}=w_{cb}$, w_{cc} are functions of $\underline{r}$ only and

$$\Phi_{ij} = (m/2k\pi T_{ij})^{3/2} \exp(-[m/2kT_{ij}][\underline{\xi}-\underline{u}_{ij}]^2) \qquad \text{(A2)}$$

(The equation for f_c is given by exchanging b and c in Eq. A1). The other parameters are given by net conservation considerations as

$$n_i = \iiint_{\Omega_i} f \; d^3\xi$$

$$2\underline{u}_{ij} = \frac{1}{n_i} \iiint_{\Omega_i} f \; \underline{\xi} \; d^3\xi + \frac{1}{n_j} \iiint_{\Omega_j} f \; \underline{\xi} \; d^3\xi \tag{A3}$$

$$\frac{6kT_{ij}}{m} = \frac{1}{n_i} \iiint_{\Omega_i} f(\underline{\xi}-\underline{u}_{ij})^2 \; d^3\xi + \frac{1}{n_j} \iiint_{\Omega_j} f(\underline{\xi}-\underline{u}_{ij})^2 \; d^3\xi$$

where Ω_i and Ω_j are the corresponding solid angles. All results presented in the main text were obtained with w set equal to unity. In this case the collision rate is independent of the relative velocity and there are some similarities to the Maxwell-Boltzmann collision operator with Maxwell molecules.

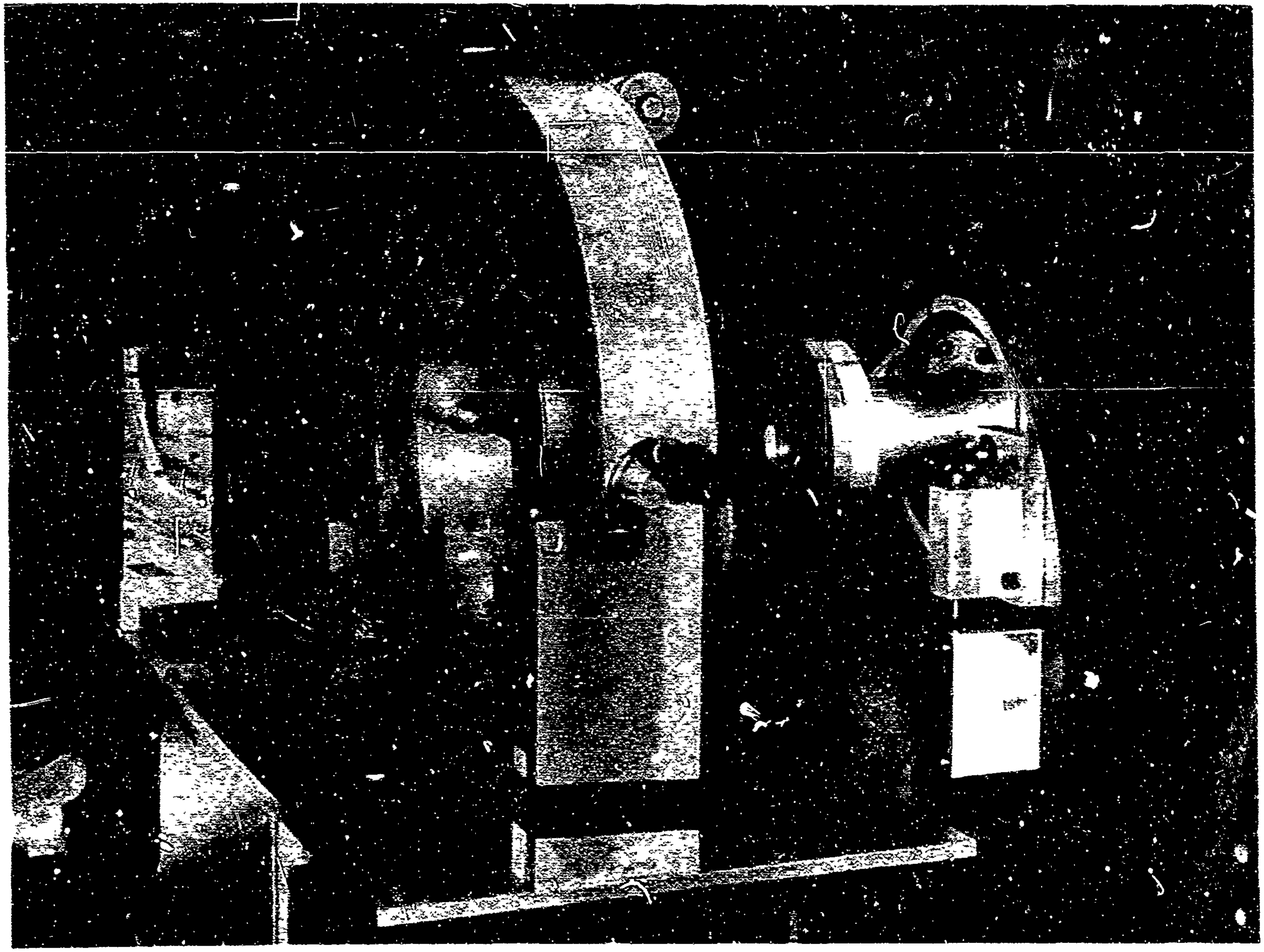

FIG. I MODEL, BALANCE AND NOZZLE IN WIND TUNNEL

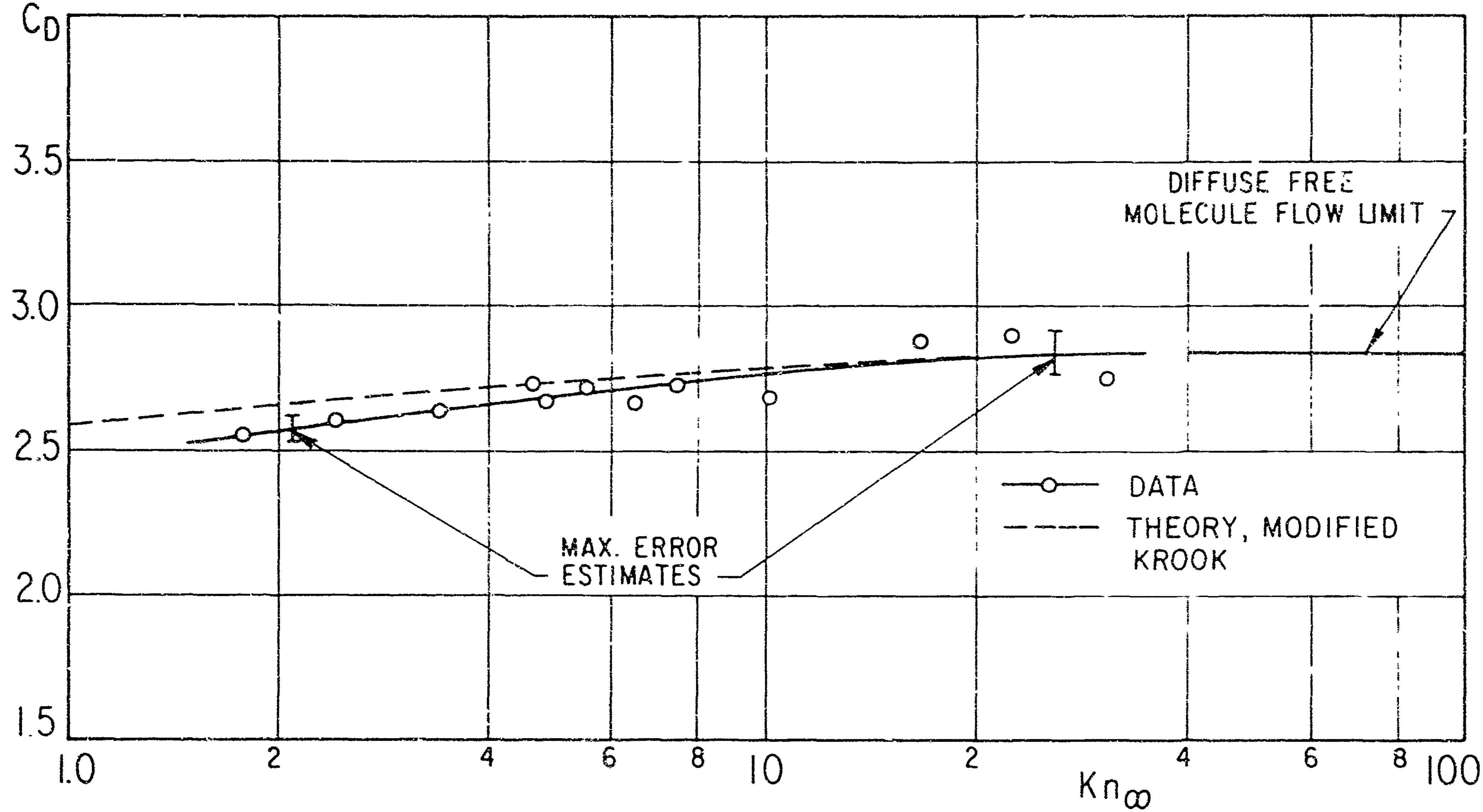

FIG. 2 CYLINDER DRAG AT M = 9.85

HYD 8085

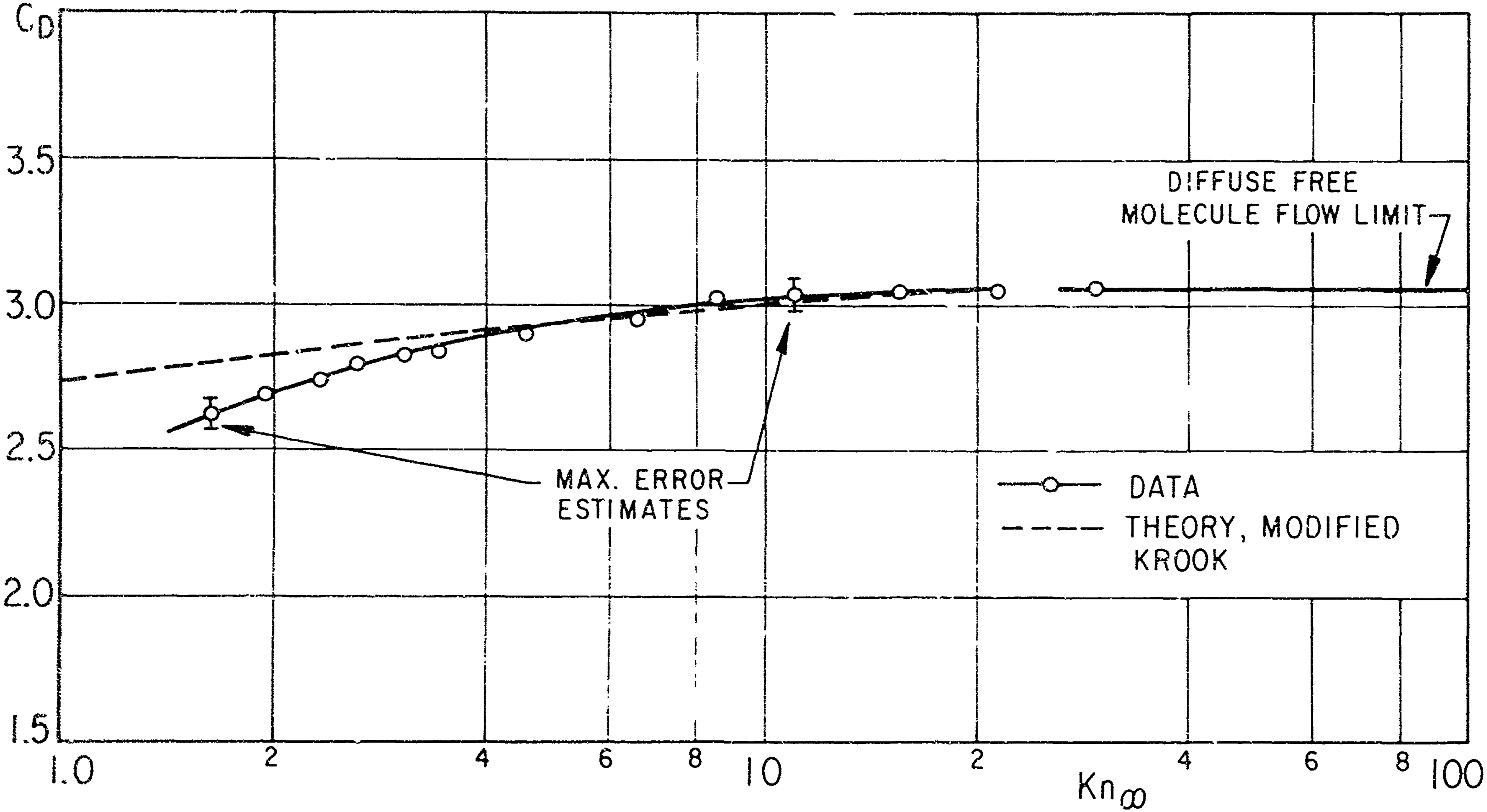

FIG. 3 DRAG OF A TWO-DIMENSIONAL STRIP AT $M \approx 10.09$

HYD 8229

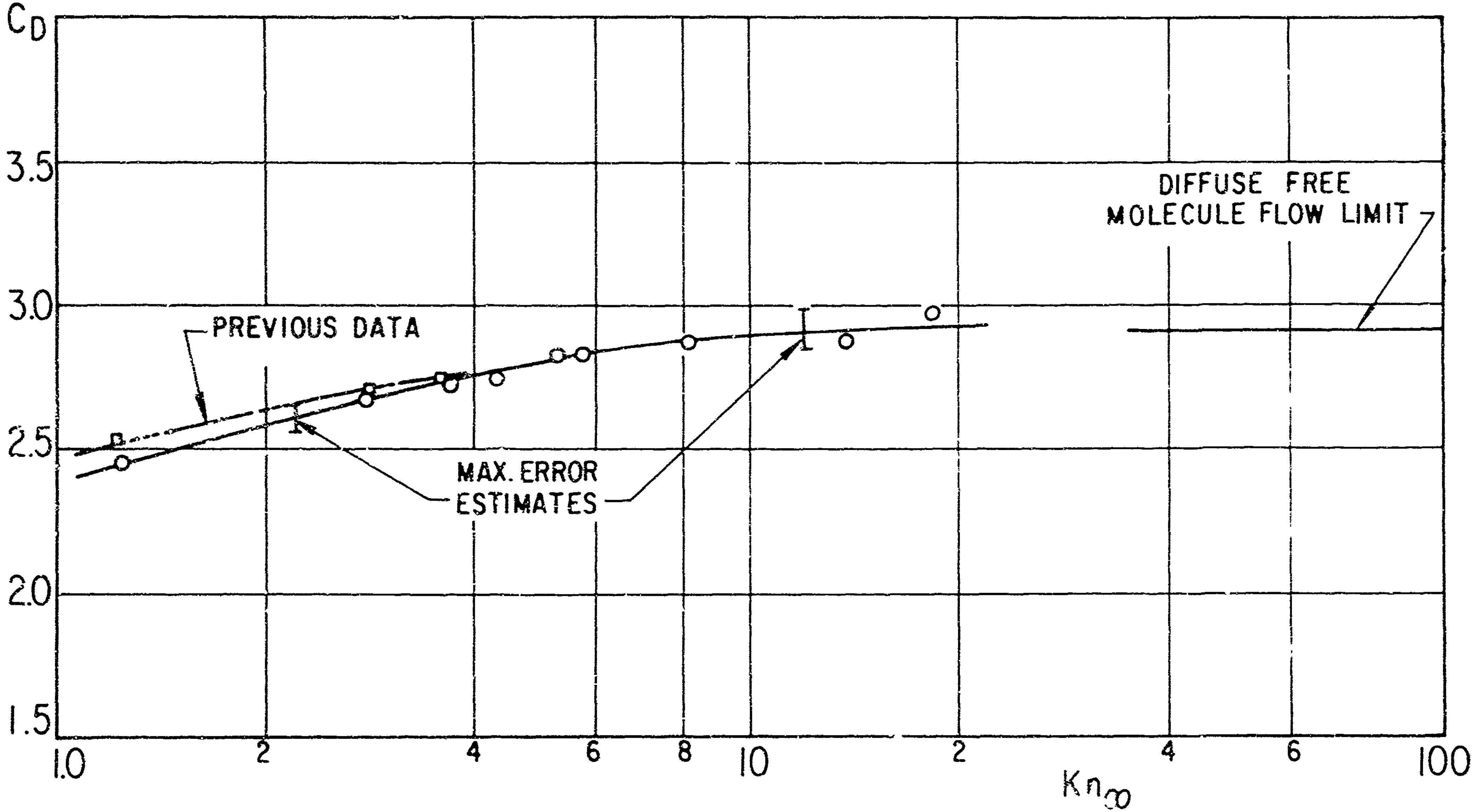

FIG. 4 CYLINDER DRAG AT M = 5.92

HYD 8082

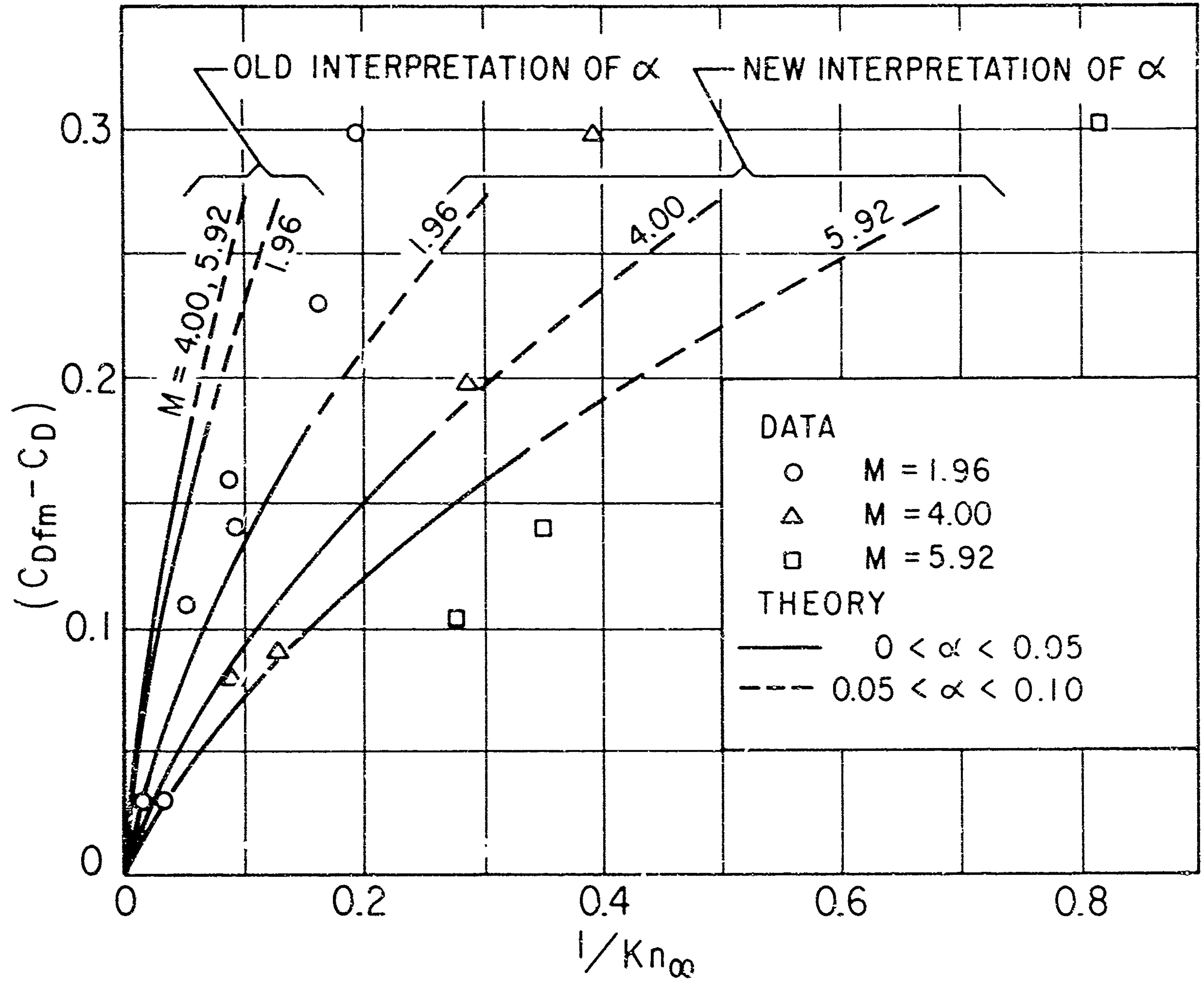

FIG. 5 COMPARISON OF THEORY AND EXPERIMENT FOR CYLINDER DRAG

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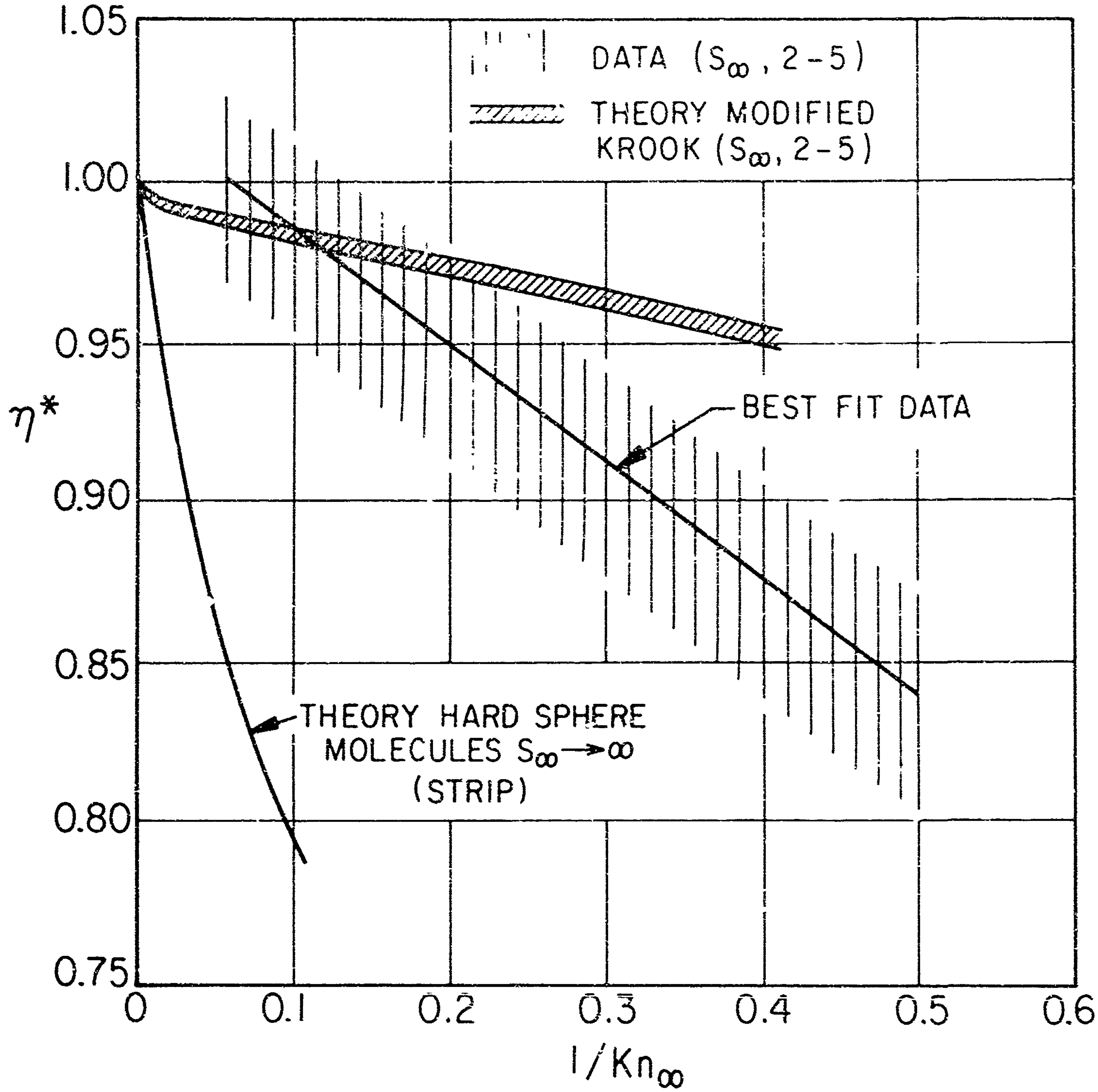

FIG. 6 COMPARISON OF THEORY AND EXPERIMENT, CYLINDER EQUILIBRIUM TEMPERATURE

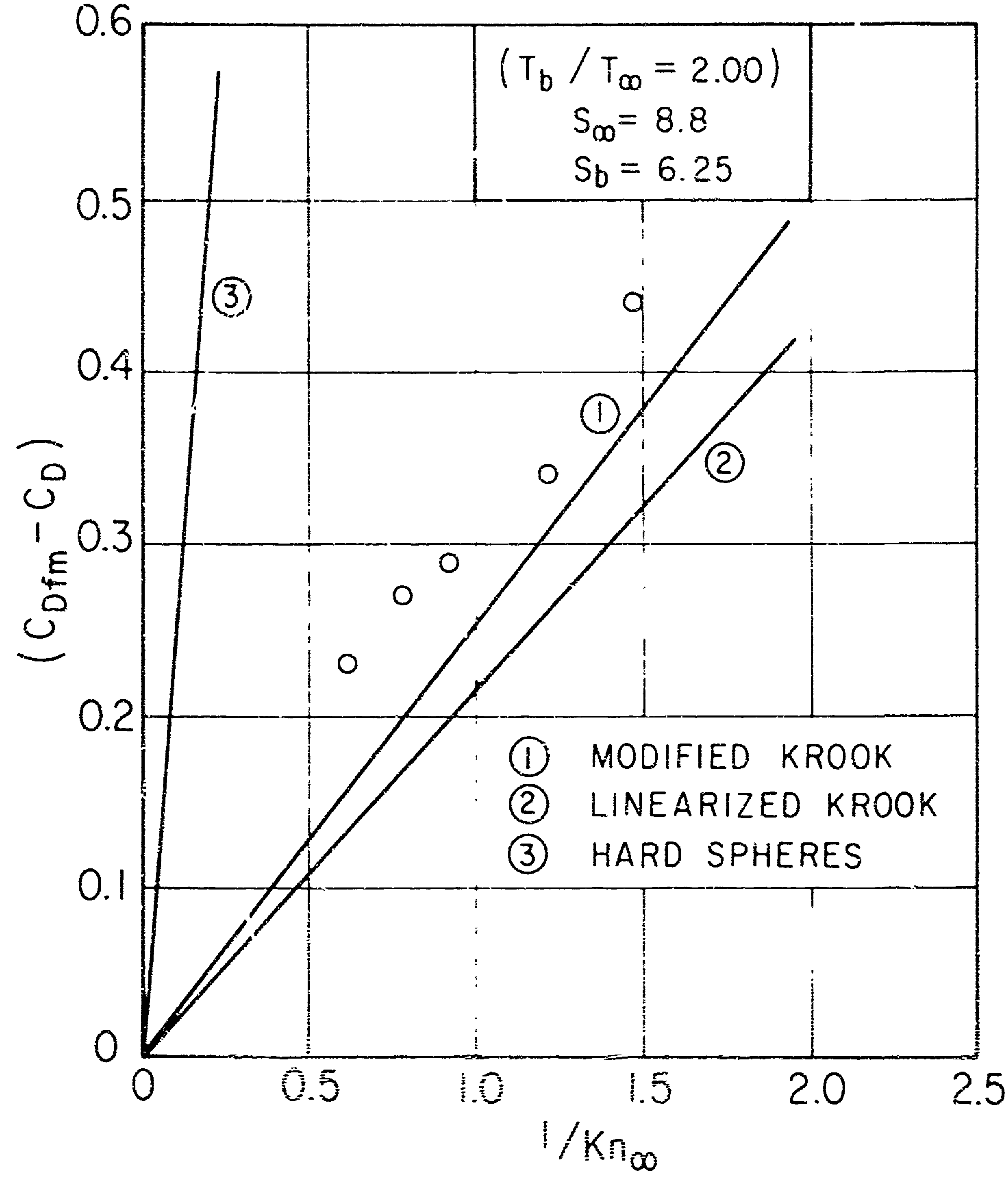

FIG. 7 COMPARISON OF THEORY AND EXPERIMENT FOR SPHERE DRAG